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From the "three-goods" macroeconomic model to the "(n+2)-goods" model: an Exploration of the Robustness of the analysis of Expectational Eductive Coordination

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From the "three-goods" macroeconomic model to the "(n+2)-goods" model : an Exploration of the Robustness of the analysis of Expectational Eductive Coordination.

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Abstract

The paper analyses the robustness of the conclusions previously obtained, showing that, in a simple three-goods model, the success of "eductive" expectational coordination relates with low supply elasticity, high demand elasticity, and high marginal propensity to consume (or high "elementary keynesian multiplier"). In a more general context, similar generalized factors have analogous qualitative effects, although new factors (heteroneity of beliefs) appear. Also, the positive coordination effects of the income effect, through the keynesian multiplier action, is now less powerful.

Résumé

Ce texte analyse la robustesse des conclusions obtenues dans un modèle classique à trois biens, montrant que la coordination divinatoire des anticipations est facilitée par une forte élasticité de la demande, une faible élasticité de l'offre et une propension à consommer élevée (ou un multiplicateur keynésien élémenntaire élevé). Dans un modéle à n+2 biens, le rôle des mêmes facteurs généralisés est souligné, bien que de nouveaux facteurs, liés à l'hétérogénéité des croyances apparaissent. L'efficacité des effets revenus, au travers du multiplicateur keynésien, est aussi atténuée.

1. Introduction.

This paper contributes to a research program that aims at assessing the Rational Expectations Hypothesis, an Hypothesis that has taken over most fields of economic modelling since the pioneering article of Muth (1961). Some skepticism on the hypothesis, as providing a uniformly valid modelling tool, motivates this program. Such feelings do not necessarily challenge the Rationality Hypothesis per se, but are at variance with Muth's original assertion according to which "the rational expectations hypothesis is nothing else than the extension of the rationality hypothesis to expectations". Such an assertion had been more or less explicitely accepted, at least untill recently, by many users of the theory. On the contrary, it is assumed here, in line with a view that is more and more frequent among economic theorists and game theorists and reflected in the just alluded to research program (see Guesnerie (2000a) for an overview), that the satisfactory coordination of expectations that occurs when they are "rational" has to be explained rather than assumed[1], and even if the individual rationality of agents is taken for granted

The critical assesment of models and hypotheses, even if they are widely accepted and routinely used, is a key activity of modern economic theory. The need and fruitfulness of such a critical effort has been taught to me by the best theorists and mathematical economists I had the chance to meet during the period of my economic education. Werner Hildenbrandt was one of them : my interest in the just evoked program reflects to some extent an influence on my intellectual development that, I hope, he would not disavow. More specifically, the present paper aims at testing the robustness of ideas that have been obtained from simpler exploratory models. I am aware that robustness tests implemented here have to be developed and completed. But I hope that the reader, and especially one of them, will be convinced that an exploration research program may still avoid to fall into "Mickey Mouse economics" !

This paper examines a stylised model of short run equilibrium, which is a multidimensional version of the model analysed in Guesnerie (2000b). It shares with the previous one its three basic features. First, firms take to-day simultaneous hiring decisions, depending on wages. Second, the profitability of these decisions depend upon to-morrow's prices and then indirectly on total income available to consumers. Third, attention is focused, along the lines of hypotheses just

[1]For example, it makes sense to adjoin the equilibrium with a process of formation of expectations. Once the expectations equilibrium is supplemented with some kind of "stability" criterion, whatever it may be, the validity of the rational expectations hypothesis is subject to verification. The success of this verification is likely to depend on the specific characteristics of the situation under scrutiny. The rational expectations hypothesis will then appear "unequally" reasonable across situations....

evoked and made fully explicit later, on the process of the formation of to-morrow expectations that determine to day's behaviour. The difference is now that the economy consists of a finite number of sectors, instead of one in the previous paper.

The plan goes as follows :

- In section 1, entitled "Preliminaries", the solution to the expectational coordination problem provided by the Rational Expectations Hypothesis is briefly and critically reassessed and the "eductive" viewpoint taken in this paper is introduced from a simple and hopefully illustrative story.
- Section 2 presents the model, the equilibrium and stability concepts.
- Section 3 gives the main results of the paper and proposes a brief discussion of its meaning and shortcomings.
- A Conclusion is finally offered.

2. Preliminaries

2.1. Justifying the Rational Expectations Hypothesis.

The usual criteria of stability of expectations belong to three broad categories.

First, the validity of the rational expectations hypothesis can be justified from the *uniqueness* of the corresponding equilibrium: a rational expectations equilibrium (R.E.E.) is an unavoidable "focal point" for the coordination of expectations whenever there is a unique such equilibrium. (A contrario, the existence of multiple REE creates ambiguity on the conditions of coordination). Insistance on uniqueness qualifies Muth's original claim, without necessarily dismissing the rationality justification that underlies it. However, modern avatars of the rationality justification have made clear that rationality alone, could not sustain the rationality of expectations and have swichted to considering whether it could derive from a much stronger assumption i.e the Common Knowledge of Rationality.

Common Knowledge of Rationality allows "understanding" or "educing" the actions taken by others, through a collective process of so-called "eductive" learning. Common Knowledge of Rationality may or may not imply the Knowledge of Equilibrium actions. The just sketched line of investigation has very close connections with the (old) game theoretical ideas that have been (recently) associated with the concepts of "rationalizable solutions", "subjectively correlated equilibrium" (but more on that later).[2]

[2] The elimination of dominated strategies is discussed already in Luce-Raiffa (1957); the cor-

A last category of justifications refers to processes of learning that are "*evolutive*" rather than "eductive": boundedly rational agents use more or less "adhoc" learning rules that reconsider expectations once they turn out to be falsified; this process may lead to the asymptotic real time convergence of the system towards the REE.

In this paper, the validity of the rational expectations hypothesis is assessed from the Common Knowledge of Rationality viewpoint. Attention is however mainly concentrated on a necessary condition, associated with the idea of Iterative Expectational stability[3], to the convergence of the "eductive "learning process triggered by the Common Knowledge Hypothesis.

It should be stressed here that there are extremely close connections between this second viewpoint, and the third "evolutive"viewpoint. In spite of the considerable differences of a priori motivations, the analysis of the consequences of Common Knowledge captures features of the situation that are also central to the convergence of evolutive learning processes.

Let us also note that our approach to the stability of expectations clearly departs from the logic of the uniqueness viewpoint, alluded to before, (in particular, because our argument is developed within a model that has a unique equilibrium). Still, it is not necessarily irrelevant to adepts of the latter viewpoint: in case of multiplicity, the type of argument used here can serve to design a selection device between equilibria.[4]

2.2. The logic of the expectational analysis :An introductory example.

Consider the following games: in Game 1, each player, in a closed room, writes down a number between zero and one hundred. The winner is the one whose number is closest to x times the average number announced by the others ($0 \leq x \leq 1$),(in case of tie, the prize is shared). Let us now consider Game 2: the rule is similar, but for two aspects: the number to announce is no longer bounded, the

responding ideas have been further elaborated by Farqhason (1969), Moulin (1979). Discussion stimulated by Bernheim (1984) and Pearce (1984) papers, includes Tan-Werlang (1988), Aumman (1987) ....

[3] à la De Canio (1978), Lucas(1978), Evans (1985). (See also, for criticisms of the foundations of the concept IE stability, that are fully answered here, Calvo (1983)).

[4] In a completely different perspective, the learning process that will be described, can also be viewed as an algorithm, which, in good cases, allows the computation of equilibrium. Within such an algorithm the questions raised by Hellwig (1993) in a similar setting, have an unambiguous, (although variable across models), answer.

coefficient x is strictly greater than one. Both games have a unique Nash equilibrium, that consists of a joint announcement of zero by all players[5]. However, several arguments sugggest that the equilibrium is more credible for Game 1 than for Game 2. The first argument can be called "eductive": in the first game, one notes that no player will play more than one hundred, so that the winning number cannot be above $100x$; but if each player realizes that, no one will announce more than $100x$, so that the winning number cannot be above $100x^2$; understanding that, nobody will announce more than $100x^2$ and so on...... But if x is strictly smaller than one, the sequence $100x^n$ converges to zero, and the speed of convergence increases when x decreases. Formally, the argument uses the fact that agents are rational (it is why they do not play more than $100x$), the fact that they know that the others are rational (so that they do not play more than $100x^2$) and ultimately, when the argument goes on, it will use the full power of the Common Knowledge of Rationality hypothesis.[6]. The conclusion of this analysis is that the Nash equilibrium of Game 1 has better justifications than the one of Game 2 [7]. One can even add that the smaller is x, the more convincing the justification is.

In a sense, the ultimate purpose of the present research program to which this paper belongs, and which has been surveyed in Guesnerie (2000a) is to transpose to the economic situations under scrutiny, each one generating one "class of game" somewhat more complex than the above one, the types of considerations that have been sketched : in some cases , one will conclude that the situation under scrutiny leads to an equilibrium that looks like the equilibrium of Game 1, when in other cases the equilibrium displays instabilities similar to those identified in Game 2. As above, at least in some cases, this basic classification will be refined by associating to the situation some number, analogous to the number x above, (negatively)correlated with the "credibility "of the equilibrium. The precise concepts used here, such as Strong Rationality (or Local Strong Rationality or Expectational Stability) of the equilibrium are briefly recalled in next Section.

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[5] This comes from the straigthforward remark that a Nash equilibrium announcement, y_1, y_2, ...,y_n, has to satisfy: $y_i = x\ (\frac{\sum_{j\neq i} y_j}{n-1})$and hence, by summation, $\sum y_i = x(\sum y_i)$.

[6] Another argument would consider the repetition of the game and the fact that boundedly rational agents will asssume, for example, that their partners write down, at step n, the winning number of step $n-1$; the repetition of the Game will bring outcomes that will come soon close to zero, and the sooner, the closer x is to zero. Naturally, both the eductive and the evolutive argument fail in Game 2: non equilibrium expectations are amplified either in a mental guessing process or in a real repetitive situation.

[7] Game 1 is one of the subjects of investigation of Nagel's thesis (1989), (1995). Her findings give much more substance to the brief assesment of the game presented here.

3. Framework and equilibria:

3.1. The basic framework.

Let us now present the general framework in which the analysis of the present text takes place. We will then be in a position to describe the equilibrium equations that will also depend on the specific market clearing assumptions that will be adopted.

The production sector consists of n sectors indexed by $l = 1, ...n$. In each sector, there are N_l firms (N_l large) that produce the same good. Firm il can hire (at most) one worker and produces α_i^l units of good l.

Without loss of generality we have $\alpha_1^l \geq \alpha_2^l \geq \alpha_3^l ... \geq \alpha_i^l \geq \alpha_{i+1}^l$ i.e firms are ranked in order of increasing productivity. The total production of good l is denoted Q_l. When firm il hires workers at the (sure) wage ω and sells its product (for sure) at price p_l, its supply- which is the walrasian competitive supply-is denoted $S_i^l(p_l/\omega, il)$.

Naturally, $S_i^l(p_l/\omega, il)$ equals one if $\alpha_i^l p_l \geq \omega$ and zero in the opposite case.

Here are then described individual decision making units: note, they are *independent decision-makers.*

One may however approximate the aggregate production function in sector l, by a smooth[8] function that relates total production of good l, Q_l to total employment E_l: $Q_l = f_l(E_l)$ and that satisfies by definition $\sum_1^n \alpha_i^l = f_l(n)$.

Total aggregate price-taking commodity supply, in sector l, $\sum_i S_i^l(p_l/\omega, il)$ is denoted $S_l(p_l/w)$.

The vector of aggregate supply is denoted $S(p/w)$.

The production sector having been defined, the basic structure of the model can be presented:

i) The model takes into account essentially two subperiods: this morning and this afternoon. In the morning, firms hire workers that are available on the labour market; in the afternoon, the goods produced from the labour hired in the morning will be sold on the goods market.

ii) Households are workers in the morning and buyers in the afternoon. Their purchasing power consists of the money holdings they carry from the day before and of the total available income (wage in the morning plus profit) they have earnt. Their desired spendings on goods depend on their prices and on available income;

[8] With indivisibilities in labour, the aggregate production would be discontinuous. With divisible labour within firm i, the function is continuous but not everywhere differentiable.

the savings behaviour implies a potential discrepancy between desired spendings and the value of production : this is a key and classical feature of the three goods models (see Barro-Grossman (1969) and for example, Malinvaud (1977)), of which this model is a $(n+2)-$ goods generalisation.

Making explicit the money holding behaviour, (and simultaneously the commodity demand behaviour) requires that we appeal to some kind of monetary theory, that will, hopefully remain simple. As we know that there is nothing like a simple and fully satisfactory monetary theory, one will have to compromise. One possibility, in the tradition of the three-goods model, is to introduce money balances in the utility function. One might also, in a similar but may be cleaner vein, adopt a variant of the cash-in-advance apparatus a la Lucas-Stokey (1988)[9].

Here, one will adopt a somewhat general formulation, a special case of which will be made fully explicit.

The general formulation consists in describing demand on the goods markets from a partly unspecified demand function $Z_l(p, R, M_0)$,when p, R, M_0,respectively designate the (n-dimensional) price vector of goods, total income, and the total quantity of money.

Note that this formulation supposes that a single index of income, R, and money holdings M_0, is a sufficient statistics for determining demand and hence, that neither the distribution of income nor the distribution of money holdings matters. As we shall see, this does not necessarily imply that we have a "representative consumer"[10]; the assumption however rules out the additional coordination problems that occur when the just evoked distributional issues matter.

Now, we shall have to assume :

A1}

i)$p.(\partial Z/\partial R)_{(.)} = \alpha^{'}(.), \alpha^{'}(.) < 1.$

ii) $p.Z_{(.)} = \alpha(.)R + M_0, \alpha(.) < 1.$

[9] Also, the model could for example be viewed as an overlapping generations model in which each time period consist of one day: then, the young work during the morning and consume and save during the afternoon.The old who have worked, consumed and saved at the previous day spend their money in the afternoon of the present day. The part of the income of the young that has not been consumed is held in money and in equilibrium equals the quantity of money previously held by the old

Although this interpretation is consistent, it tells a story that is unlikely to make clear sense when the period length has to be viewed as short (much below half a life!)...

[10] In particular, the implicit model has a large number of consumers and even when they are identical, the analysis would not view them as representative, since differences of beliefs are always a priori considered.

The marginal (i) and the average (ii) propensity to consume (from current income) are smaller than one. Also ii) says that previously saved money is used to consume.

Also,we shall often assume for the local analysis of expectational coordination, that :

A2$\}\alpha(.) = \alpha'(.) = Cste$

The appendix des cribes a fully specified prototype of the model, where money holdings are explained with a very crude signal story[11]. The advantage of this crude signal story is that the intertemporal equilibrium reduces to a sequence of "daily" equilibria : the intertemporal "eductive" problem, that we are going to analyse, identifies with the intra day problem. In the more sophisticated version of the model, for example when money appears in the utility function, to-day demand of money holdings depends on price expectations to-morrow. Rational expectations means rational expectations within the day and rational expectations across days.

The inconvenience of the story is that the demand effects associated with a proportional increase of all commodity prices are too simplistic : they reduce to an income effect, ruling out important substitution effects between goods and money balances[12]

3.2. "Daily" Equilibria.

The market clearing assumptions can now be made explicit :

1) a market clearing price equalizes the supply of the firms and competitive demand of households.

2) the money wage in the morning is fixed at $\overline{\omega}$

Finally, we suppose that the morning labour supply is not responsive to prices and is fixed (later E_0)

We define accordingly for the considered day[13], a fixed wage (keynesian?) equilibrium, in which agents, in the morning have rational expectations.

[11] Note also that the explicit protoype model always fit assumption A1) and fits assumption A2), whenever the two types of consuming agents have the same marginal propensity to save : $\{\partial D_{Cl}/\partial R\}(p, R) = \{\partial D_{Pl}/\partial R\}(p, R + M_O/(1-\alpha))$,

In that case, the $\alpha()$ of the general formulation is nothing else than the α of the special model.

[12] Note that such substitution effects are allowed to play a role in Guesnerie (2000a).

[13] I use here the word daily rather than temporary that migth be misleading : temporary equilibria are associated with exogenous expectations. .

Definition 3.1. *A "Daily" Rational Expectations (Keynesian) Fixed Wage Equilibrium of this system, the total quantity of money being M_0, will be associated with production levels $Q_l^\star$, prices $p_l^\star$, national income $R^\star$ that will meet the following conditions:*

i) $Z_l(p^*, R^*, M_0) = Q_l^*, \forall l = 1, ...n.$

ii)$R^* = \sum_l p_l^* Q_l^*$

iii)$S_l(p^*/\overline{\omega}) = Q_l^*$

iv)$\sum_l f_l^{-1}(Q_l^*) < E_0$

The equations speak for themselves :

i) expresses market clearing on all the goods markets, and because of A1-ii) implies market clearing on the money market.[14]

ii) says that total income, wage plus profit is equally attributed to consumers.

iii) is the production supply when the price vector is $(p^*, \overline{\omega})$ and iv) says that total labour demand from the firms is smaller than labour supply.

Again, the modelling options are those of the so-called three goods model, but adapted to the n-commodity context under consideration : the simplifying formulation of excess demand has already been discussed : its one-dimensional counterpart fits the formulation of Guesnerie (2000 b). The implicit (to ii)) assumption of uniform labour rationing seems reasonable for our purpose; the fact that the total income is available in the afternoon, rather than wage income for example, reflects a standard although debatable modelling choice, but which has only secondary effects on our analysis (see Guesnerie (2000b) for a more comprehensive discussion).

3.3. Eductive Coordination"

The model has then, flexible prices, fixed wage and rational expectations. The first two assumptions are taken for granted, the third one is not. More precisely, the question under scrutiny here is whether the equilibrium is Strongly Rational in sense of Guesnerie (1992) (one could still say that it is the unique rationalizable

[14] In our specific model, Equation i) can be made explicit as i')

i$'$)$(2\alpha - 1)D_{Cl}(p^*, R^*) + (1 - \alpha)D_{Pl}(p^*, R^* + M_0/(1 - \alpha)) = Q_l^*, \forall l = 1, ...n.$

The money market equilibrium can be commented in a more explicit way : $\alpha\mu = \alpha(2 - 1/\alpha)$ agents consume to-day, from their to-day income, $(1 - \alpha)$ consume to-day from their to-day income plus the money acquired yersteday ($M_O/(1 - \alpha)$ per capita); (i$'$) plus ii), together with the definition of D, and the fact that $\alpha(1 - \mu) = 1 - \alpha$ save the whole income R^*, imply that the money market clears.

equilibirum or that it is "dominant solvable"). The concept and a number of its applications have been discussed elsewhere (see for example Guesnerie (2000a)). Roughly speaking, we wonder whether the assumption that morning decision-makers, i.e here the firms, have Common Knowledge of the situation triggers the conclusion that they have Common Knowledge (from now on CK) of the equilibrium, or in other words whether the iterative elimination of non best responses trigerred by the CK knowlege assumption leads to the equilibrium situation : when the answer is positive, we say that the equilibrium is Strongly Rational. We however limit attention to the case where initial beliefs are restricted (at least hypothetically) by a local Common Knowledge Restriction; the corresponding concept is that of local Strong Rationality.

Let us sum up : our analysis a priori focuses attention on the relationship between the following assertions :

A1) In the morning :

- Profit maximising firms are Bayesian Rational.

- They know that the morning wage is fixed at $\overline{w}$,and that the market clearing prices to-morrow will be determined from total supply Q_l in all sectors, according to the following equations :

$$Z_l(p, R, M_O) = Q_l$$

- These two facts are Common Knowledge.

A2) The sequence $(Q_1.....Q_k)$ belongs to a compact, of non empty interior , neighbourhood of $(Q_1^*.....Q_k^*)$, the fixed wage Rational Expectations Equilibrium and this fact is (hypothetically)[15] CK

A3) The fixed wage Rational Expectations Equilibrium $(Q_1^*.....Q_k^*)$, is CK.

When A1)$\Longrightarrow$A3), the equilibrium is Strongly Rational.

When A1) plus A2) $\Longrightarrow$A3), for some small neighbourhood, the Equilibrium is said Locally Strongly Rational.

In fact the paper mainly focuses on a weaker condition called Iterative Expectational Stability, which is necessary for Strong Rationality and that we shall

[15] The reason why this restriction is said to be hypothetical, unless it were implemented by some appropriate external policy (import or export call for example), is that A1) may be in a sense incompatible with A3) : this will occur whenever actions compatible with A1) will trigger quantities outside of the considered neighbourhood. In a sense, Local Strong Rationality (LSR) only asserts that A2) is a coherent sentence, at least for some non trivial neighbourhood. It would probably be logically more satisfactory, but plausibly less appealing, to introduce LSR in this way.

explain below.

4. Necessary conditions for "eductive coordination": one result.

4.1. Preliminaries.

Let us remind that if the production of good k is denoted Q_k , then the market clearing equations for goods can be written:

$$Z_l(p_1, p_2, ..., p_n; \sum_k p_k Q_k; M_0) = Q_l$$

Differentating the above expression, it comes :
$\sum_k \{\partial Z_l/\partial p_k + Q_k \partial Z_l/\partial R\} dp_k + (\sum_{k \neq l} \{p_k \partial Z_l/\partial R)\} dQ_k) = (1 - p_l \partial Z_l/\partial R) dQ_l$
With vector notation:

$$(\partial Z)\, dp = (I - A) dQ$$

where (∂Z) is a matrix of pseudo-compensated[16]demand and where A is the matrix whose diagonal elements are of the form $(p_l \partial Z_l/\partial R)$ and off diagonal elements are of the form $(p_k \partial Z_l/\partial R)$ in the lth row, kth column.

Let us say a few words on the matrix A :

$$\begin{pmatrix} p_1\partial Z_1/\partial R & p_k\partial Z_1/\partial R & .... & .... & p_n\partial Z_1/\partial R \\ .... & .... & .... & ..... & .... \\ p_1\partial Z_l/\partial R & p_k\partial Z_l/\partial R & p_l\partial Z_l/\partial R & ... & p_n\partial Z_l/\partial R \\ .... & .... & .... & ..... & .... \\ p_1\partial Z_n/\partial R & p_k\partial Z_n/\partial R & .... & .... & p_n\partial Z_n/\partial R \end{pmatrix}$$

A is of rank one : all its columns are proportional. Its image is one-dimensional, along the direction of the vector $\partial Z/\partial R$. Note that $(\partial Z/\partial R) = \alpha^{'}$ (resp.α), where α' (resp.α) is also the marginal (average) propensity to consume, so that $\alpha^{'}$(resp.α) is the eigenvalue associated with the eigenvector $\partial Z/\partial R$: Indeed, $\alpha^{'} < 1$, or with

[16] In the more detailed formulation, it is the sum of the matrix of compensated like (indeed the matrix of compensated derivatives in the interpretation taken here, where demand conditional to the signal C comes from a continuum of identical consumers) derivatives associated with the demand triggered by the signal C, and of the Jacobian matrix associated with D_P ,the demand trigerred, to-day, by the signal P, yersteday.

assumption A2)[17], $\alpha < 1$, is the only non zero eigenvalue, so that the matrix A is productive. Also, its kernel is $n-1$ dimensional : $\{x/p.x = 0\}$[18]

Now, let us come to the matrix $I - A$:

$\partial Z/\partial R$ is an eigenvector associated with the eigenvalue $1-\alpha$, and the space normal to p is an eigenspace with eigenvalue 1.

4.2. A basic insight.

From now, unless explicit mention of the contrary, we shall assume that A1) and A2) hold true, and for their local part at the equilibrium. (Note that nothing crucial depends on A2), but it makes the reference to the specific model immediate). We can state :

Proposition 4.1. *The Fixed wage Rational Expectations Equilibrium is Locally Strongly Rational only if it is Locally Iteratively Expectationally Stable.*

When (∂Z) is invertible, this latter fact is equivalent to :

The eigenvalue of highest modulus of the matrix $(\partial S)(\partial Z)^{-1}(I-A)$, evaluated at equilibrium, is smaller than one.

Proof.

We first introduce informally the matrix $(\partial S)(\partial Z)^{-1}(I-A)$. A formal definition of Iterative Expectational Stability, as well as a proof of the fact that it is required for Local Strong Rationality, are given in Evans- Guesnerie (1993). For the first part of the Proposition, it has only to be shown that the present model can be encompassed in their setting, so that their Proposition 2 applies.

Here, we shall stick to an unformal argument and definition, that makes however clear why the matrix $(\partial S)(\partial Z)^{-1}(I-A)$ matters

Intuitively, Strong Rationality implies the following : If all agents, here firms, have beliefs over the production vector, associated with probability distributions the support of which is a small compact ball around the equilibrium productions,

[17] (i.e in the explicit formulation with identical income effects across consumers)

[18] Note that the transpose of A, A^t is :

$p_1\partial Z_1/\partial R)$	$p_1\partial Z_l/\partial R)$	....	....	$p_1\partial Z_n/\partial R)$
....	....	....	.....	....
$p_l\partial Z_1/\partial R)$	$p_l\partial Z_k/\partial R)$	$p_l\partial Z_l/\partial R$	...	$p_l\partial Z_n/\partial R)$
....	....	....	.....	....
$p_n\partial Z_1/\partial R)$	$p_n\partial Z_l/\partial R)$	....	....	$p_n\partial Z_n/\partial R)$

It has proportional lines, an eigenvector p, (with eigenvalue α under the conditions stressed above) and a kernel : $\{x/(\partial Z/\partial R).x = 0\}$

then it must intuitively be the case that, for Strong Rationality to hold, either local or global, the individual decisions trigerred by such beliefs induce total production levels that remain close to the equilibrium quantities, or in the case under consideration, remain in the initial ball.

A fortiori, the above remark applies whenever the initial beliefs of the firms are degenerate, for example when they are identical and consists of point expectations, rather than of probability distributions : it is indeed what the concept of Iterative Expectational Stability expresses (see footnote 3).

Then, let us consider point expectations that differ from the equilibrium expectations by

$dQ = (dQ_1, dQ_2, ...dQ_n)$,

The corresponding belief triggers, according to the above market clearing equation , and assuming that (∂Z) is invertible a change in prices equals to :

$dp = (\partial Z)^{-1} (I - A)dQ$

a price change that induces a supply change :

$(\partial S) (\partial Z)^{-1} (I - A)dQ$

For the evoked condition to hold, the associated linear mapping must be contracting :

Hence the second part of the proposition.

■

The qualitative factors that are stressed here are the same as those appearing in the case of a single commodity treated in Guesnerie (2000b). Indeed, with one comodity, $(\partial S) (\partial Z)^{-1} (I - A)$ is nothing else, with straightforward notation, than $(s/z)(1-\alpha)$, which is the expression found in the just quoted article[19], i.e the ratio of the derivative of supply with respect to price and of the derivative of demand multiplied by the marginal propensity to save. Equivalently, what matters is *the ratio of supply elasticity over demand elasticity multiplied by the inverse of the elementary keynesian multiplier.* The n-commodities analysis makes however clear that the qualitative conclusions of the 1-commodity case have to be qualified.

- As we said, in the n-goods case, the condition stressed here is a necessary and sufficient condition for Local Iterative Expectational Stability, but contrarily to what happens in the three-goods case, this condition is not equivalent to Local Strong Rationality[20].

[19] However, in the formulation of the standard three goods model, the marginal propensity to consume is allowed to differ from the average one, contrarily to what has been assumed here.

[20] Note that the argument requires efficient labour rationing, an assumption that is not used here.

- In the general case, a necessary and sufficient condition for Local Strong Rationality would have to take into account additional restrictions associated with the heterogeneity of beliefs : however, the analysis of this dimension of expectational coordination would routinely proceed along the lines of Evans-Guesnerie (1993).

5. Further Inquiry :

Let us summarize.

i) Unsurprisingly, the heterogeneity of beliefs has a potentially more destabilizing effect on expectational coordination, in an n-dimensional world than in a one-dimensional world, a fact that is reflected in the fact that IE Stability is no longer sufficient for Strong Rationality.

ii) Leaving in the shadow the heterogenity of beliefs, one also concludes that the qualitative features of the one-dimensional analysis of Iterative Expectational Stability have a n-dimensional close counterpart : the counterpart of the supply and demand derivatives are, the supply and demand Jacobian matrices ∂S and ∂Z, the counterpart of the (marginal) propensity to save - or of the elementary Keynesian multiplier - are $I - A$, or $(I - A)^{-1}$. However, it is unclear to which extent the powerful effects on expectational coordination of a small propensity to save, or equivalently a high "elementary" Keynesian multiplier, generalize here.

Let us clarify this latter point : the one dimensional formula : $(s/z)(1 - \alpha) < 1$, does not provide, strictly speaking comparative statics results[21], unless we compare situations with constant elasticities of supply and demand and fixed propensity to consume. However, it stresses a useful and clear fact that will be a key role in any comparative statics exercise : everything equal, a smaller propensity to consume, or a higher "elementary keynesian multiplier" favours stability. To which extent is such a qualitative fact still true in the general case ?

The role of the keynesian effects, stressed in simple game theoretical models with strategic complementarities by Cooper and John (1989), has been ascertained in the more complex context of the three-goods model, in which strategic complementarities are dominated by strategic substitutabilities, (Guesnerie (2000b). The question just raised is then a robustness issue. Can we still clearly detect and ascertain, in the n-goods model, and within the same pre-comparative statics

[21] It is interesting to compare the "eductive stability" of (equilibria of) different economies or, in the same economy, of exogenous changes or different policy choices.

exercise performed in the three-goods case, a positive expectational coordination effect of the marginal propensity to consume[22]?

A first attempt at clarifying these issues is presented now.

The basic norm inequality shows that a sufficient condition for IE Stability is $\|\partial S\| \left\|(\partial Z)^{-1}\right\| \|(I-A)\| < 1$.

We would provide a clearcut and strong extension of the one-dimensional case, if we could replace the inequality by $(s^{\max}/z^{\min})(1-\alpha) < 1$, where $s^{\max}, z^{\min}, \alpha$ are respectively the maximal supply derivative, the modulus of the eigenvalue of smallest modulus of (∂Z) , and the propensity to consume.

Clearly, this cannot be exactly true : for any norm, it follows from Horn-Johnson(1991) that $\left\|(\partial Z)^{-1}\right\| \geq 1/z^{\min}$, but also, and more disturbingly, that $\|(I-A)\| \geq 1$.

However, one can still exploit the freedom in the choice of norms, ($\|.\|$ designated *any* a priori chosen matrix norm), together with the properties of A, in order to get some additional insights on the conditions of IE-Stability.

Note, for example, that the first assertion is equivalent to :

$\|\partial S\| \left\|(\partial Z)^{-1}\right\| < 1/\|(I-A)\|$

and $\|(I-A)\| \leq \|I\| + \|A)\|$

Now let us remind that the spectral norm of A is the absolute value of the eigenvalue with highest absolute valueof $A^t A$ (Horn-Jonhson (1991), p.295).

Now, the symmetric matrix $A^t A$, consists of proportional lines, the lth line being, with straigthforward notation : $\{(\partial Z/\partial R)^2 p_l\} p^t$, and has a unique non zero eigenvalue $(\partial Z/\partial R)^2 (p)^2$, associated with the eigenvector p.

It follows that the spectral norm[23] of A is $\|p\| \|\partial Z/\partial R\|$, where $\|.\|$ designates the Euclidean vector norm. Writing $p.\partial Z/\partial R = \ \|p\| \|\partial Z/\partial R\| (\cos\theta)$, it follows that the spectral norm of A is $\alpha/(\cos\theta)$.

This proves :

When $\|.\|$ is the spectral norm, a sufficient condition for IE-Stability, is :

$\|\partial S\| \left\|(\partial Z)^{-1}\right\| < 1/(1+\|A\|) = \alpha/(\alpha+\cos\theta)$, *for some "angle" θ between p and $\partial Z/\partial R$.*

In fact, the just obtained statement is not conclusive : a higher marginal propensity to consume does not seem to make, everything equal, the sufficient

[22] In a sense, the mathematical question that is associated might be formulated as follows : Assume that, as we shall do later, the matrix A is parametrized by β, with $A_\beta = \beta A$. How does the modulus of the eigenvalue of highest modulus of $(\partial S)(\partial Z^{-1})(I - A_\beta)$ vary with β ?

[23] As a matrix norm, then the spectral norm it is higher than the modulus ot the eigenvalue of highest modulus of A.

condition easier to check, on the contrary ![24]

The above analysis of sufficiency is disappointing for our purpose and suggests, to the least, that the role of the marginal propensity to save is now much more ambiguous. We are however, now, exhibiting a necessary condition, that provides a clearer link with the one dimensional case.

Proposition 5.1. *For IE Stability to hold, it is necessary that :*

$(\frac{1}{1-\alpha})\,\|(\partial Z)\|\,\|(\partial S\ )^{-1}\| > 1$

where $\|.\|$ *is the matrix norm induced by the Euclidean vector norm on a basis of* R^n *that consits of the vector* $(\partial Z/\partial R)$ *and of an orthogonal basis in the hyperplane normal to* p.

Proof. For IE stability to hold, it is necessary that the modulus of the eigenvalue of highest modulus of the matrix $B = (\partial S)\,(\partial Z)^{-1}\,(I - A)$ be (strictly) smaller than 1[25]. This implies that the modulus of all eigenvalues of $B^{-1} = (I-A)^{-1}\,(\partial Z)\,(\partial S)^{-1}$ is (strictly) higher than 1, and consequently that the norm of B^{-1}, for any matrix norm is greater than one :

$\|(I-A)^{-1}\,(\partial Z)\,(\partial S)^{-1}\| > 1$

But $\|(I-A)^{-1}\|\,\|(\partial Z)\|\,\|(\partial S\ )^{-1}\| \geq \|(I-A)^{-1}\,(\partial Z)\,(\partial S)^{-1}\|$

Also, since A is productive, $(I-A)^{-1} = I + A + A^2 + ...$ and $\|(I-A)^{-1}\| = \|I + A + A^2 + ..\|$

and$\|I\| + \|A\| + \|A^2\| + ... \|A^n\| + ... \geq \|I + A + A^2 + ..\|$

so that, necessarily :

$(\|I\| + \|A\| + \|A^2\| + ... \|A^n\| + ...)\,\|(\partial Z)\|\,\|(\partial S\ )^{-1}\| > 1$

Now take as matrix norm, the norm induced by the Euclidean norm taken on the basis associated with the eigen space of A, that consists of the vector $\partial Z/\partial R$, and of any basis of the hyper plane orthogonal to p.

With this matrix norm :$\|I\| = 1, \|A\| = \alpha$.

Hence the conclusion.

■

The necessary condition of the proposition is interesting, in particular because its one dimensional counterpart reduces to the necessary and sufficient condition of

[24] Also, the relationship between the type of norms adopted and the eigenvalue of highest modulus (for example, the spectral norm of (∂Z) and its spectral radius) is a priori unclear and cannor be fully elucidated in the absence of more specific assumtions on demand.

[25] One says also that the *spectral radius* is smaller than one (see Horn-Johnson (1991)

Guesnerie (2000b). Also, the role of the elementary keynesian multiplier $\frac{1}{1-\alpha}$,[26]in trigerring the above general necessary condition justifies comments similar to the qualitative comments made on the three goods-model.

Another teaching of the analysis is that, in order to fully elucidate the exact interplay of income and price effects in expectational coordination, we probably need :

- A more careful assesement of the properties of excess demand .
- A preliminary assesement of the product $(\partial Z)^{-1}(I-A)$, as a whole, as opposed to the previous separate assesment of (the norms of) the two matrices (∂Z), $(I-A)$.

The full exploitation of these remarks is outside the scope of the present analysis. We shall however explore the case where excess demand meets the gross substitutability assumption, so that $-(\partial Z)^{-1}$ is a positive matrix. Hence $-(\partial S)(\partial Z)^{-1}$ is a positive matrix.

Let us define, as in footnote 22, $A_\beta = \beta A$, for positive β. This provides a parametrisation of the marginal propensity to consume which equals $\beta\alpha$ for the matrix A_β

There is then a range of value of $\beta, (0, \beta^{\max})$ for which

$-B_\beta = -(\partial S)(\partial Z)^{-1}(I - A_\beta) = -(\partial S)(\partial Z)^{-1} + (\partial S)(\partial Z)^{-1} A_\beta$

is a positive matrix.

We show :

Proposition 5.2. *Under the gross substitues assumption introduced above, if or some* β_0, $(\partial S)(\partial Z)^{-1}(I - A_\beta)$ *has a spectral radius smaller than one, then the spectral radius of decreases in the range* $\beta \in (\beta_0, \beta^{\max})$.

Proof. We note, from Perron-Frobenius, that the eigenvalue of highest modulus is real and positive and smaller than one for $-B_{\beta_0}$. The fact that this eigenvalue decreases for $\beta > \beta_0$, reflects the fact that if $\beta \in (\beta_0, \beta^{\max})$, $-B_{\beta_0} > -B_\beta$. Then, when lim $(-\lambda B_{\beta_0})^n, \lambda > 0$, tends to zero, there exists $\gamma > 1$, such that lim $(-\gamma\lambda B_\beta)^n$. The conclusion follows from the fact that limit $A^n = 0$ is equivalent to the fact that the spectral radius of A is strictly smaller than one (Horn-Johnson (1991). ■

As emphasized previously, the result is not, strictly speaking , a comparative statics result : in a complex problem, when one changes β, one also changes

[26]Obviously, however,$\|\partial Z\|$ is higher than the spectral radius of ∂Z. (the same remark applies to $(\partial S\ )^{-1}$)

$(\partial\ S)$ and (∂Z). However, it unambiguously stresses the positive effect of the "elementary keynesian multiplier" on coordination, agreeing with the intuition drawn from the one-dimensional analysis.

6. Conclusion:

This paper has attempted to check the robustness of the expectational analysis of a three good-type model by imbedding it in an n-dimensional context. It has confirmed the role and the qualitative effects of supply and demand derivatives in expectational coordination. However the stabilizing role of income effects is likely, "in general" to be less clearcut in the n-dimensional context considered here than what was suggested by the one dimensional analysis of the three goods model. The full clarification of the problem calls for additional work. Such work is likely to stress specificities of expectational coordination that relate with the specific forms of the global Excess demand function : it may then have at this stage closer connections with the work of Werner Hildenbrandt on consumption.

Note, finally that related papers study expectational coordination along similar lines in general equilibrium : in particular, Ghosal (1999) and Guesnerie-Hens (2000) focus attention on a two-period exchange economy, and provide complementary pieces for a more comprehensive theory of expectational coordination in standard general equilibrium models.

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APPENDIX

7.1. A fully specified special case.

The specific story that provides a fully explicit and parcimonious illustration of the general model,[27] is the following. There is a continuum of agents of mass one. At each period, each agent receives a signal, either C, consume immediately, or P, postpone consumption next period. These signals are i.i.d between agents and periods and the probability of C is μ. The signals are obeyed - for example an agent who receives C consumes to-day its available income - but in case of contradiction : an agent who has received P yersteday, consumes to-day, whatever signal he receives to-day[28]. In the steady state of the process, a proportion α of agents consume with $1-\alpha+\alpha\mu=\alpha$, i.e $\alpha = 1/(2-\mu)$[29]. Equivalently, $\mu = (2-1/\alpha), 1-\mu = 1/\alpha - 1$

In this crude version, the individual demand is necessarily of the form $D_S(p,R)$[30], where p is the vector of goods prices (expressed in terms of money, the price of which is one) and R is the income available to the agent, and where S designates either C or P, (immediate consumption behaviour may be different from postponed consumption behaviour).

Under this assumption, aggregate demand in the afternoon is nothing else, (with obvious notation), than :

$\int_C D_C(p,R_h)dR_h + \int_{Ps} D_P(p,R_h)dR_h$

The following analysis is however much simplified if we assume that demand does not depend on the income distribution. This occurs if, for example all agents are a priori similar. Then, aggregate demand equals :

$\alpha D_C(p,R) + D_P(p,.)$

where R is the total income of an agent who has received the signal C, and . is the income of an agent who have received P.

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[27] For a similar story in a finance context, see Allen-Gale (1994).

[28] Equivalently, the process C,P, can be viewed as a two states Markov process with the transition matrix : $\begin{matrix} \mu & 1 \\ 1\text{-}\mu & 0 \end{matrix}$ and the ergodic distribution $(\alpha, 1-\alpha)$

[29] For example for $\mu = 0, \alpha = 1/2$, half of the agents consume to-day, because they have not consumed to- morrow, half the agents postpone, because they obey their signal. With $\mu = 0, \alpha = 1$, all agents receive the C signal and consume.

[30] The story can however be made more sophisticated, by supposing that the signal affects positively or negatively the rate of time preference between to-day and to-morrow : money holdings serve to adapt consumption to signals, and the aggregate demand function is more complex that previously.